Transition of Signals from Time Sampled Form to Frequency Sampled Form using Transformation Methods

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Highlights

- Time Sampled Signals
- Freq. Sampled Signals
- Transformation

Received date: 13-01-23
Accepted date: 03-02-23
Published date: 08-03-23

Abstract: This research paper deals with the transformation of digital signals from time domain representation to the frequency domain representation. This transformation is completed in three phases, the first phase is transformation from digitally time domain represented signal to sampled stripe set based signal through time sampling procedure and the second phase is transformation from time sampled stripe set based signal to frequency domain represented signal with certain angular frequency and the third phase is from frequency domain represented signal to sampled stripe set formed frequency domain based signal through frequency sampling procedure. The necessity for signal transformation is the industry need for utilization of digital signal processing based devices in wireless communication systems. The models of transformation of digital signals from time domain representation to frequency domain in sampled form are proposed systematically with using classical digital signal processing models. After using and evaluating the mathematical models I got that for odd and fractional values of k, the Fourier transforms of digital signals have real and imaginary part whereas for even values of k, the Fourier transforms of digital signals have only real form.

Keywords: Time sampling; frequency sampling; digital transformation; Discrete Fourier Transforms; Fourier Transforms; and Z- transforms.

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1. Introduction

When we discuss wireless communication systems, it is indispensable for us to elaborate the mathematical nature of signals processed through wireless communication systems. Generally a telecommunication system consists of transmitter, receiver, line of sight and medium of transmission. When signals are transmitted from transmitter side to the receiver side, the signals are passed through many phases of conversion from one form to another form. On the front side the signals are transmitted, processed and received and on the back side the signals are mathematically represented and processed and converted from one form to another form. The signals are of different type and following different medium but signal representation in different medium is of utmost importance. The medium of transmission of signals can be air, copper wire, coaxial cable and water. Depending upon the medium of transmission, the channel capacity, signal transmission bandwidth and signal to noise ratio varies. Although the medium of transmission of signals has effects on signals transmission, time domain representation and frequency domain representation is of significance from the signal sampling and representation perspective as discussed in [1].

In time domain representation of signals the signals are represented in time domain and over a complete time span the signals appearance, form and shape is expressed graphically as referred in [4]. Whereas in frequency domain the signals are represented in frequency domain and over a complete frequency span the signals appearance, form and shape is expressed graphically. I can elaborate the frequency domain representation of signals in two signals form namely the discrete time signals and continuous time signals. The discrete time signals are measured and represented in vertical stripe sets form and having either zero value or one value as referred in [3]. Whereas the continuous time signals are measured in frequency domain and these signals have angular frequency domain representation over a fixed span of angular frequencies. The time domain representation of signals and frequency domain representation of signals both are used depending upon the requirement of system through which the signals are processed. Depending upon the type of system in which the signals are processed, the signals are processed and according to the necessary requirement of the system, the signals are converted from time domain to the frequency domain as referred in [7].

Generally when we talk about signal processing or digital signal processing, the system which process the signals is related with the type of signals and signals representation in either time domain or frequency domain. Also there is always a need of conversion from time domain representation to frequency domain representation or vice versa depending upon the need of system which can be either analog or digital. Also there exists such devices used specifically for time domain representation of signals and frequency domain representation of signals. The analysis of signals in time domain and frequency domain is dependent upon the need of the system requirements as mentioned in [7].

The devices used for signal transformation from time domain to frequency domain representation are the oscilloscopes or spectrum analyzers shown here in figure 1.

Figure 1. Devices for signals analysis (**a**) Oscilloscope; (**b**) Spectrum Analyzer.

2. Research Methodology

The research methodology for the proposed work is quantitative. The mathematical equations for the signal transformation from time domain to frequency domain representation are proposed in this research paper and the transformation method is evaluated in the numeric form. The results derived from the proposed quantitative method are discussed further in the research paper.

3. Background and Literature Review

In this research paper, I will discuss the signals representation in time domain as well as the signals representation in frequency domain and will elaborate on the conversion of signals from time domain to frequency domain for usability in digital signal processing systems.The conversion from time domain to frequency domain or from frequency domain to time domain is of major significance. The scientific reason for conversion of time domain signals to frequency domain signals or from frequency domain signals to time domain signals is that in some systems there is a need of time domain representation of signals and in some systems there is a need of frequency domain representation of signals or when there is a transition of time domain based systems to frequency domain based systems. Therefore different scientists worked on the conversion techniques from time domain representation to frequency domain representation as discussed in [7].

In traditional communication systems, sampling of digital signals is discussed in more detail as referred by Carlson in an authoritative text Communication Systems referred in [5]. When signals are represented in time domain, there is a need of sampling of signals because the time sampled signals are processed in more efficient manner in frequency domain representation. Sampling is defined as getting stripe set of the given time domain signal so that the retrieved stripe set can be sampled and used in frequency domain representation. Mathematically sampled signals are represented by 'Ts' and 'n' is the number of samples in time sampled signal and 't' is the time during which the digital signal exists in time domain.

The journey of transformation from time domain signals to frequency domain signals start from the sampling procedure of time domain based signals as referred in [2]. Since the characteristics of digital signals are more understandable in time domain like periodicity as referred in classical text of digital signal processing by Proakis as referred in [6].The periodicity of digital signals is defined as the recurrence of digital signal after a specific period of time. The transformation of digital signals from the time domain to the frequency domain with reference to periodicity is of interest as future work.

Referring [7] the time sampled signal is not directly transformed from time domain to frequency domain , but there exist an intermediate transitionary phase of transformation in which 'n' stripe sets of sampled signals are developed and processed after sampling procedure and then these 'n' stripe sets of sampled signals are converted into frequency domain representation through three transformation methods.

Generally speaking the transformation of time domain represented signal to frequency domain represented signal involves three phases. The first phase incorporates transformation from digital signal in time domain representation to time sampled digital signal through sampling method. The second phase incorporates transformation from time sampled digital signal to frequency domain represented digital signal. The third phase incorporates transformation from frequency domain represented digital signal to frequency sampled signal through frequency sampling method.

4. Proposed Models for Signal Transformation

Suppose that x(t) represents the digital signal in time domain representation, Ts represents the sampling time of digital signal, t represents the total time period of digital signal in time domain, n represents the number of stripe sets, x(n) represents the time sampled signal with n stripe sets x(z) represents the digital signal in Z domain, X(jw) represents the digital signal in frequency domain, X(k) represents the digital signal sampled in frequency domain with k frequency samples.

The total time period for existence of digital signal in time domain is given as

$$\boldsymbol{t = n\ \ Ts} \tag{1}$$

The angular frequency of the given digital signal is represented with the following equation

$$\boldsymbol{w = \frac{2\pi k}{N}} \tag{2}$$

The digital signal time sampled with n stripe sets is assumed here by equation 3.

$$\boldsymbol{x(n) = (0.4)^n a(n)} \tag{3}$$

The general formula for transformation to Z transform is given by equation 4.

$$\boldsymbol{a^n a(n) \rightarrow \frac{z}{z-a}} \tag{4}$$

The above mentioned equations 1 and 2 are the fundamental equations for time sampling and the angular frequency. Using equation 4 we get the Z transformed function of the given digitally time stamped signal as given below:

$$\boldsymbol{X(z) = \frac{z}{z-0.4} \times \frac{z^{-1}}{z^{-1}}}$$

(5)

Now simplifying the given function in 5 we get

$$\boldsymbol{X(z) = \frac{1}{1-0.4z^{-1}}}$$

(6)

Transforming Z transform to the frequency domain representation of the digitally time stamped signal, we get

$$\boldsymbol{z = re^{jw}}$$

(7)

Assume that the constant has 1 value:

$$\boldsymbol{r = 1}$$

(8)

Substituting equation 7 and equation 8 in equation 6 we get the Fourier Transform of the given digital signal in time domain:

$$\boldsymbol{X(jw) = \frac{1}{1-0.4e^{-jw}}}$$

(9)

Assume that the sampling frequency for transformation is 4 as given by equation 10:

$$\boldsymbol{N = 4}$$

(10)

The angular frequency for 4 stripe sets of the given digital signal will be given by equation 11:

$$\boldsymbol{w = \frac{2\pi k}{4}}$$

(11)

Using equation 4 as

$$\boldsymbol{a^n a(n) \rightarrow \frac{z}{z-a}}$$

The Euler's Identity will be given by equation 12:

$$e^{j\theta} = \cos \theta + j\sin\theta \quad (12)$$

The Euler's Identity will be given by equation 13:

$$e^{-j\theta} = \cos \theta - j\sin\theta \quad (13)$$

Let

$$\theta = \frac{\pi}{2} \rightarrow e^{\pm j\frac{\pi}{2}} \quad (14)$$

Then

$$\theta = \frac{\pi}{2} \quad (15)$$

By substitution in Euler's identity

$$e^{-j\frac{\pi}{2}} = \cos \frac{\pi}{2} - j \sin \frac{\pi}{2} \quad (16)$$

Since

$$\cos \frac{\pi}{2} = 0 \quad (17)$$

And

$$\sin \frac{\pi}{2} = 1 \quad (18)$$

Then

$$e^{-j\frac{\pi}{2}} = -j \quad (19)$$

The frequency domain representation of the given digital signal in time domain is represented by equation 20:

$$X(k) = \frac{1}{1 - 0.4e^{-j\pi k/2}} \quad (20)$$

Assume 4 values of sampling frequency

$$k = 0, 1, 2, 3 \quad (21)$$

We get

$$\boldsymbol{X(0) = \frac{1}{1 - 0.4e^{-j\pi(0)/2}}}$$

(22)

Taking

$$\boldsymbol{k = 0}$$

(23)

Then

$$\boldsymbol{X(0) = \frac{1}{1 - 0.4e^{-j\pi(0)/2}}}$$

(24)

Solving

$$\boldsymbol{X(0) = \frac{1}{1 - 0.4e^{0}}}$$

(25)

$$\boldsymbol{X(0) = \frac{1}{0.6}}$$

(26)

Taking

$$\boldsymbol{k = 1}$$

(27)

Then

$$\boldsymbol{X(1) = \frac{1}{1 - 0.4e^{-j\pi(1)/2}}}$$

(28)

Simplifying

$$\boldsymbol{X(1) = \frac{1}{1 + 0.4j} \times \frac{1 - 0.4j}{1 - 0.4j}}$$

(29)

$$\boldsymbol{X(1) = \frac{1 - 0.4j}{1^2 + 0.4^2}}$$

(30)

$$X(1) = \frac{1 - 0.4j}{1 + 0.16}$$

(31)

$$X(1) = \frac{1}{1.16}(1 - 0.4j)$$

(32)

Taking

$$k = 2$$

(33)

We get

$$X(2) = \frac{1}{1 - 0.4e^{-j\pi\frac{2}{2}}}$$

(34)

$$X(2) = \frac{1}{1 - 0.4e^{-j\pi}}$$

(35)

$$e^{-j\pi} = cos\ \ \pi - jsin\ \ \pi$$

(36)

$$e^{-j\pi} = \ -1 - j(0)$$

(37)

$$e^{-j\pi} = \ -1$$

(38)

$$X(2) = \frac{1}{1 - 0.4(-1)}$$

(39)

$$X(2) = \frac{1}{1 + 0.4}$$

(40)

$$X(2) = \frac{1}{1.4}$$

(41)

Taking

$$\boldsymbol{k = \frac{3}{2} = 1.5}$$

(42)

We get

$$\boldsymbol{X(3/2) = \frac{1}{1 - 0.4e^{-j\pi\frac{3}{4}}}}$$

(43)

$$\boldsymbol{e^{-j\pi\frac{3}{4}} = cos\frac{3}{4}\ \ \pi - jsin\frac{3}{4}\ \ \pi}$$

(44)

$$\boldsymbol{e^{-j\pi\frac{3}{4}} = \ \ -\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}}$$

(45)

$$\boldsymbol{X\left(\frac{3}{2}\right) = \frac{1}{1 - 0.4(-\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2})}}$$

(46)

$$\boldsymbol{X\left(\frac{3}{2}\right) = \frac{1}{1 + j0.2\sqrt{2} + 0.2\sqrt{2}}}$$

(47)

$$\boldsymbol{X\left(\frac{3}{2}\right) = \frac{1}{1 + 0.2j\sqrt{2}}}$$

(48)

$$\boldsymbol{X\left(\frac{3}{2}\right) = \frac{1}{1 + 0.2j\sqrt{2}} \times \frac{1 - 0.2j\sqrt{2}}{1 - 0.2j\sqrt{2}}}$$

(49)

$$\boldsymbol{X\left(\frac{3}{2}\right) = \frac{1 - 0.2j\sqrt{2}}{1^2 + \ \ (0.2\sqrt{2})^2}}$$

(50)

$$\boldsymbol{X}\left(\frac{\mathbf{3}}{\mathbf{2}}\right) = \frac{\mathbf{1 - 0.2}\boldsymbol{j}\sqrt{\mathbf{2}}}{\mathbf{1 + (0.04 \times 2)}}$$

(51)

$$\boldsymbol{X}\left(\frac{\mathbf{3}}{\mathbf{2}}\right) = \frac{\mathbf{1 - 0.2}\boldsymbol{j}\sqrt{\mathbf{2}}}{\mathbf{1.08}}$$

(52)

$$\boldsymbol{X}\left(\frac{\mathbf{3}}{\mathbf{2}}\right) = \frac{\mathbf{1}}{\mathbf{1.08}} - \frac{\mathbf{0.2}\boldsymbol{j}\sqrt{\mathbf{2}}}{\mathbf{1.08}}$$

(53)

Here we took sampling frequency of 4 and transformed the time sampled digital signal from time domain to Discrete Fourier Transformed form of the signal in frequency domain.

3. Results

After evaluating the mathematical transformation from time domain to frequency domain representation of digital signals, we got the following results:

Table 1. The table consists of the values of index k and sampled values X (k)

Number	k	X(k)
1	0	1/0.6
2	1	1/1.16(1-0.4j)
3	1.5	1/1.08(1-.2828j)
4	2	1/1.4

4. Discussion

In this research paper, the proposed method for the transformation of signals from time domain to frequency domain can be implemented through MATLAB software using the relevant transformation commands. The results show that the frequency sampled form of signals may contain real values as well as complex values.

5. Conclusions

As far as our quantitative research is concerned, we derived that for odd and fractional values of k, the Fourier transforms of digital signals have real and imaginary part whereas for even values of k, the Fourier transforms of digital signals have only real form.

Author Contributions: Conceptualization, Ahmed Faraz and Mrs. Shumaila Usman.; methodology, Ahmed Faraz; software, Ahmed Faraz; validation, Ahmed Faraz, and Mrs. Shumaila Usman.; formal analysis, Ahmed Faraz; investigation, Ahmed Faraz; resources, Ahmed Faraz; data curation, Ahmed Faraz; writing—original draft preparation, Ahmed Faraz; writing—review and editing, Ahmed Faraz; visualization, Ahmed Faraz; supervision, Ahmed Faraz; project administration, Ahmed Faraz.

Funding: This research received no external funding.

Acknowledgments: Special thanks to Director General Bahria University Karachi Campus for provision of support and motivation for this research work.

Conflicts of Interest: We declare no conflict of interest.

References

1. Khan, M., Hasnain, S. K., & Jamil, M. "Digital Signal Processing: A Breadth-first Approach". CRC Press, 2022
2. Li, L., Han, N., Jiang, Q., & Chui, C. K. "A chirplet transform-based mode retrieval method for multicomponent signals with crossover instantaneous frequencies". Digital Signal Processing, 120, 103262, 2022
3. Cuisano, J., Flores, F., Chirinos, L., & Vaudrey, A. "In-cylinder pressure statistical analysis and digital signal processing methods for studying the combustion of a natural gas/diesel heavy-duty engine at low load conditions ". Energy Conversion and Management, 269, 116089, 2022
4. Pukhova, V., Gorelova, E., Ferrini, G., & Burnasheva, S. "Time-frequency representation of signals by wavelet transform". In 2017 IEEE Conference of Russian Young Researchers in Electrical and Electronic Engineering (EIConRus) (pp. 715-718). IEEE, 2017, February
5. Carlson, A. B., & Crilly, P. B. "Communication Systems", 5e. 2010
6. Proakis, J. G., Salehi, M., Zhou, N., & Li, X. "Communication systems engineering (Vol. 2)". New Jersey: Prentice Hall. 1994
7. Yang, L., Chen, J., Wang, S., & Gao, J. "Dielectric response measurement of oil-paper insulation based on system identification and its time-frequency-domain conversion method". IEEE Transactions on Dielectrics and Electrical Insulation, 25(5), 1688-1698, 2018